

2020

## MATHEMATICS — HONOURS

Paper : DSE-A-1

(Industrial Mathematics)

Full Marks : 65

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.*

1. Choose the correct answer with proper justification / explanation for each of the multiple choice question given below : (For each question, one mark for each correct answer and one mark for justification) : 2×10
- (a) In the CT scan, we use... beams to detect the suspected broken bone locations within the medium.
- (i) Hard X-ray (ii) Soft X-ray  
 (iii) Electron (iv)  $\gamma$ -ray.
- (b) Differential equation known as Beer's law is an
- (i) ordinary 2nd order linear differential equation  
 (ii) ordinary 2nd order nonlinear differential equation  
 (iii) ordinary 1st order linear differential equation  
 (iv) ordinary 1st order nonlinear differential equation.
- (c) The definition of a periodic function, is given by a function which
- (i) has a period  $T = 2\pi$  (ii) satisfied  $f(t + T) = f(t)$   
 (iii) satisfied  $f(t + T) + f(t) = 0$  (iv) has a period  $T = \pi$ .
- (d) A signal  $x(t)$  has a Fourier Transform  $X(\omega)$ . If  $x(t)$  is real and odd Function of  $t$ , then  $X(\omega)$  is
- (i) a real and even function of  $\omega$   
 (ii) an imaginary and odd function of  $\omega$   
 (iii) an imaginary and even function of  $\omega$   
 (iv) a real and odd function of  $\omega$ .
- (e) A line  $\mathcal{L}_{t,\theta} = \{(t \cos \theta - s \sin \theta, t \sin \theta + s \cos \theta) : -\infty < s < \infty\}$  is perpendicular to the unit vector  $\mathbf{n}$ . Then
- (i)  $\mathbf{n} = (\cos\theta, \sin\theta)$  (ii)  $\mathbf{n} = (-\cos\theta, \sin\theta)$   
 (iii)  $\mathbf{n} = (\cos\theta, -\sin\theta)$  (iv)  $\mathbf{n} = (-\cos\theta, -\sin\theta)$ .

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(f) The value of the integral  $\int_{-\infty}^{\infty} e^{-Ax^2} dx$  is

(i)  $\frac{\pi}{A}$

(ii)  $\sqrt{\frac{\pi}{A}}$

(iii)  $\frac{1}{A}$

(iv)  $\frac{1}{\sqrt{A}}$ .

(g) If  $\delta(x)$  be a delta function, such that  $\int_{-\infty}^{\infty} \delta(x)dx = 1$ , then the Fourier transform of  $\delta(x)$  is

(i) 1

(ii)  $\frac{1}{\delta(1)}$

(iii)  $\delta(1)$

(iv)  $\sqrt{\delta(1)}$ .

(h) If the  $2 \times 2$  matrix  $X$  satisfies the equation  $X \begin{pmatrix} 4 & 7 \\ 5 & 9 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix}$ , then  $X =$

(i)  $\begin{pmatrix} -6 & 4 \\ 13 & -10 \end{pmatrix}$

(ii)  $\begin{pmatrix} -6 & 5 \\ 13 & -10 \end{pmatrix}$

(iii)  $\begin{pmatrix} -6 & 4 \\ 12 & -10 \end{pmatrix}$

(iv)  $\begin{pmatrix} -6 & 4 \\ 13 & -1 \end{pmatrix}$ .

(i) If  $\mathcal{R}f(t, \theta)$  denotes the Radon transform of  $f$ , which one of the following is true?

(i)  $\mathcal{R}(\alpha f + \beta g) = \alpha^2 \mathcal{R}f + \beta^2 \mathcal{R}g$

(ii)  $\mathcal{R}(\alpha f + \beta g) = \alpha \mathcal{R}f + \beta \mathcal{R}g$

(iii)  $\mathcal{R}(\alpha f + \beta g) = (\alpha-1)\mathcal{R}f + (\beta-1)\mathcal{R}g$

(iv)  $\mathcal{R}(\alpha f + \beta g) = \mathcal{R}f + \mathcal{R}g$ .

(j) If  $f$  is continuous on the real line,  $\int_{-\infty}^{\infty} |f(x)| dx < \infty$  and  $\mathcal{F}$  denotes the Fourier transform of  $f$ , then

(i)  $\mathcal{F}^{-1}(\mathcal{F}f)(x) = f^{-1}(x) \forall x$

(ii)  $\mathcal{F}^{-1}(\mathcal{F}f)(x) = f^2(x) \forall x$

(iii)  $\mathcal{F}^{-1}(\mathcal{F}f)(x) = 2f(x) \forall x$

(iv)  $\mathcal{F}^{-1}(\mathcal{F}f)(x) = f(x) \forall x$ .

### Unit - I

2. Answer **any two** questions :

(a) In CT scan which kind of X-ray is used and why? Explain with suitable example. 5

(b) (i) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = x^3 + 1$ . Find  $(f^{-1})'(28)$ .

(ii) Find all complex numbers  $z$  such that  $|z| = 1$  and  $|z^2 + \bar{z}^2| = 1$ . 2+3

(3)

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- (c) If  $A$  be a real matrix, then prove that all the eigenvalues  $A^T A$  are non-negative real numbers and the corresponding eigenvectors are orthogonal. 5
- (d) Solve the differential equation  $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = x^2 e^x$ . 5

**Unit - II**

3. Answer **any two** questions : 5×2

- (a) What do you mean by an inverse problem of a mathematical problem? Explain it with an example.
- (b) Write down the inverse problem of the direct problem : Compute the eigenvalues of the given matrix  $A + D$ , where  $A$  being a real symmetric matrix of order  $n \times n$  and  $D$  is a  $n \times n$  diagonal matrix.

- (c) Find the eigenvalues and the corresponding eigenvectors of the matrix  $A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & -1 \\ 3 & 2 & -2 \end{pmatrix}$ .

- (d) Solve the differential equation,  $\frac{dy}{dx} + \frac{x}{1-x^2} y = x\sqrt{y}$ .

**Unit - III**

4. Answer **any one** question : 5×1

- (a) State Beer's law on X-ray beam. Write its differential equation form. Establish the result

$$\int_{x_0}^{x_1} A(x) dx = \ln \left( \frac{I_0}{I_1} \right)$$

where  $A(x)$  is the attenuation coefficient function and  $I(x)$  is the intensity of the X-ray beam.

- (b) An X-ray beam  $A(x)$ , propagates in a medium is defined by

$$A(x) = \begin{cases} 1 - |x|, & \text{if } |x| \leq 1, \\ 0, & \text{if } |x| > 1 \end{cases}$$

Find the intensity  $I(x)$  of this beam, with the initial condition  $I(-1) = 1$ .

**Unit - IV**

5. Answer **any one** question : 5×1

- (a) Find the Random transform of the function

$$f(x, y) = \begin{cases} 1 - \sqrt{x^2 + y^2}, & \text{if } x^2 + y^2 \leq 1 \\ 0 & \text{if } x^2 + y^2 > 1 \end{cases} \text{ on a line } \mathcal{L}_{t,0}.$$

- (b) Write a short note on Shepp-Logan Mathematical phantom.

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**Unit - V**

6. Answer **any one** question :

- (a) Define back projection. Prove that the back projection is a linear transformation. 2+3
- (b) Give an example of back projection in the context of medical imaging. 5

**Unit - VI**

7. Answer **any two** questions : 5×2

- (a) Write a short note on CT scan within 500 words.
- (b) Describe an algorithm of CT scan machine.
- (c) Find the Fourier transformation of the function  $(ax^2 + bx + c)e^{-dx^2}$ ,  $-\infty < x < \infty$ , where  $a, b, c, d > 0$ .

(d) If  $f$  be a continuous functions, such that  $\int_{-\infty}^{\infty} |f(x)| dx < \infty$ , then prove that  $\mathcal{F}^{-1}(\mathcal{F}f)(x) = f(x)$  for all

$x$ , where  $\mathcal{F}f$  and  $\mathcal{F}^{-1}f$  denote respectively the Fourier and inverse Fourier transform of  $f$ .

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